

The Effect of Singapore
Mathematics on Student
Proficiency in a Massachusetts
School District: a Longitudinal
Statistical Examination

Executive Summary

The Gabriella and Paul Rosenbaum Foundation

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Executive Summary

Implementation of Singapore Math – Conclusions

The results from our analyses show that:

- a) participation in SM classes has a positive impact on student MCAS test scores,
- b) the duration of student participation in SM classes has a greater positive impact on test score gains than SM participation at any particular grade level and
- c) beginning SM in early grades improves the curriculum's effectiveness.

Introduction

North Middlesex Regional School District (NMRSD) is the only Massachusetts school district with Singapore mathematics (SM) for its curriculum. Since it is also the American school district with the longest period of SM implementation, its student outcomes over an extended period afford exceptional opportunities for a longitudinal statistical study.

At the same time that NMRSD was establishing its SM curriculum, Massachusetts was in the process of implementing ambitious academic goals. Mathematics standards (based in part on Singapore's), tests and teacher certification requirements were all progressively toughened, to considerable effect. Participating in TIMSS 2007 as a separate State, Massachusetts earned 5th place overall, solidly ahead of most of the participating countries including the U.S. itself. Thus, NMRSD's SM implementation was taking place under challenging conditions.

Approximately 5,000 pre-K to 12 students are enrolled in NMRSD's 4 elementary, 2 middle and 1 high school. SM implementation was gradual, beginning in the 2000-01 school-year with 6 SM classrooms. By 2005-06, all 93 of NMRSD grades 1 through 6 classrooms were SM classes. Grades 7 and 8 reached 100% SM participation in 2007-08.

Source/Verification of the Data Employed by the Study

The Singapore mathematics textbook series used by NMRSD schools¹ differ from textbooks used in all other Massachusetts (MA) school districts. The State's student population at each grade is about 70,000 students, while

¹ *Primary Mathematics, 3rd edition* (and later, *U.S. edition*) were used in grades 1-6. *PM Standards Edition*, aligned with California Standards which are similar to MA Standards, has not yet been used by NMRSD since it was only published in 2008. Grades 7-8 used *New Elementary Mathematics (Syllabus D)*.

NMRSD's is about 300 students. Thus, a reasonable approach in assessing the effect of Singapore math on NMRSD students is through comparison of their mathematical performance with that of all of MA students. And since NMRSD's SM implementation took place over several years, it was also possible to compare SM versus non-SM student performance within the District itself, as well as with the state as a whole. The Massachusetts Comprehensive Assessment System (MCAS) which evaluates student, school, and district mathematics performance is an ideal assessment tool.²

Comparisons with MA

Comparing the math performance of NMRSD's students with that of MA's students overall reveals NMRSD advantages. For all MCAS tests given during 2003 through 2008 Table 1 (p. 8) lists the percent correct MCAS scores for MA's students, for NMRSD's students, and the difference between their scores. It can be seen that NMRSD student scores are higher than those of MA students, in all but 3 of the 24 grade-years. These results are significant by analysis of variance (ANOVA) ($F=56.069$, $P<0.001$, $df = 1, 32$).

State test results are generally presented in summary performance level categories: "Advanced", "Proficient", "Needs Improvement" and "Warning / Failing". Comparisons between NMRSD and MA results can be made using these categories, just as effectively as using actual MCAS scores.

For example see Table 2 (p. 9), the χ^2 analyses comparing MA and NMRSD categorical³ scores for all grades tested in 2006. This shows that:

- a) For all grades 4 to 8 and 10, the differences in NMRSD and MA categorical results are statistically significant. That is, NMRSD results for grades 5, 6, 7, 8 and 10 are significantly better than MA's.
- b) An exception: MA's grade 4 results are better than NMRSD's, particularly in the years 2006-2008.

While NMRSD's 3rd grade results are also better than MA's, this difference is not statistically significant.

There is another advantage to categorical results. They clarify the two kinds of good NMRSD results:

- a) The sum of Advanced and Proficient categories is greater for NMRSD than for MA.
- b) The sum of Needs Improvement and Warning/Failing categories is lower for NMRSD than for MA.

² During school years 1998-1999 and 1999-2000, MCAS tested only grades 4, 8 and 10; 2000-01 to 2004-05, grade 6 tests were added; only since 2005-06 have grades 3 to 8 and 10 been tested.

³ While these are technically interval variables, the interval changes year-by-year according to scaled-score results and we prefer to treat them as categorical.

For example for 6th grade, NMRSD has 56% \geq Proficient and 44% < lower than Proficient compared to 46% and 54% for MA. These 2006 results are typical, with NMRSD usually performing significantly better than MA for most grade-years.

Longitudinal Comparisons — Experience with Singapore Math

To illustrate the effect of SM on individual students, we followed students over the period 2002-08 and found their scores increasing with increased CSM.

Students were grouped by grade-year into four cohorts, each with over 1,000 student-trials. This provides for a very powerful study.

Each cohort met three conditions.

- a) All students within a cohort were in the same grade in the same school-year.
- b) All of their MCAS math scores for 2002 through 2008 were available.
- c) All cohort students had a verifiable CSM of 3 or more years.

The progression of the cohorts through their schooling is considered as vectors over time. For example, cohort V4 was tested five times during grades 4 through 8, with V4's student-trials ranging from 357 in 2005 to 345 in 2009 for a total of 1,711 student-trials.

The results of regression analysis for our cohorts, with students' MCAS scores as the dependent variable and CSM as the independent variable, are statistically significant ($t=3.537$, $P=0.001$, coefficient 2.070).

When further controlled for student qualifiers (Free-lunch and Special Education)⁴, MCAS scores remain significantly positively related to CSM ($t=2.231$, $P=0.026$, $\text{coeff.}=0.200$).

These results may be seen graphically in Figure 2 (p. 11) which shows the number and frequency of MCAS scores (as a percent correct) attained by SM and non-SM students. As can be seen, the percentage of non-SM students who have low scores is greater overall: the majority do not score above or even at the level of mean NMRSD scores.

Perhaps most telling: Table 3 on page 12, where results for cohort V1 students (those who were in 4th grade in 2002) are directly related to CSM. The efficacy of SM is shown – and, the table may also illustrate the inefficacy of leaving SM instruction until later grades.

⁴ Notably, Special Education students are shown to benefit from CSM as well, with mean MCAS scores of 26, 27, 27, 28, 28 and 33 for 0 through 5 years of CSM, respectively.

This latter point may be clearer if we look at 8th grade scores from 2006 by CSM – see Table 4 on page 13. Students with CSM of 2 years – the same students with CSM of 0, 0, 2 years in Table 3 – score below students with 3 or 5 years of CSM. Interestingly, the scores of students with 4 years of CSM are about the same as students with 2 years of CSM. Referring again to Table 3, we see that these 4-year-CSM students are overwhelmingly those students who missed SM in 4th grade.

Effect of Teachers' Experience with SM on MCAS Performance

Figure 3 (p. 14) summarizes NMRSD students' math performance in all their MCAS-tested grades during 2002-08, separated according to their teachers' SM experience (0 to 4 or more years) at each testing time.

Table 5 on page 15 provides the differences in MCAS mean scores achieved by cohort students whose instructors taught 60 or more students. Most are positive and all of the students of teachers with more than one class without SM show improvement with SM.

Finally, it may be instructive to quote from Mary Waight's 2006 testimony to the National Math Panel⁵:

Improving outcomes for students in mathematics is dependent on a number of factors, chief among them a teacher with a strong math background, ongoing professional development, administrative support and involvement, and a mathematics program that encourages mathematical understanding. North Middlesex seems to have found the answer.

⁵ Waight, M.M., *The Implementation of Singapore Mathematics in a Regional School District in Massachusetts: 2000-2006*, in *Remarks to a National Mathematics Advisory Panel*. 2006: Cambridge, MA.[]

Appendix

Figures and Tables

Table 1. Coded (Mean and Adjusted Mean) MCAS scores by grade by year				
Year	Grade	MA Mean (% correct)	NMRSD (% correct)	Δ (NMRSD minus MA)
2003	4	63	68	5
	6	60	64	4
	8	55	63	8
	10	54	58	4
2004	4	65	70	5
	6	66	72	6
	8	58	64	6
	10	65	73	8
2005	4	68	71	3
	6	62	70	8
	8	57	61	4
	10	65	75	10
2006	3	78	79	1
	4	71	70	-1
	5	64	68	4
	6	66	71	5
	7	61	68	7
	8	62	67	5
	10	65	74	9
2007	3	75	80	5
	4	70	68	-2
	5	67	69	2
	6	67	70	3
	7	65	69	4
	8	62	68	6
	10	67	72	5
2008	3	75	90	15
	4	68	67	-1
	5	66	66	0
	6	72	74	2
	7	66	68	2
	8	63	66	3
	10	66	73	7

The Effect of Singapore Mathematics on Student Proficiency in a Massachusetts School District

Table 2. χ^2 Analyses for 2006, MA and NMRSD, by Grade (Numbers are percent of students in each category)						
2006 Grade		Advanced	Proficient	Needs Improvement	Warning/ Failing	
3	MA	4	48	32	16	$\chi^2= 0.766$ (P = 0.857)
3	NMRSD	4	47	34	15	
4	MA	15	25	45	15	$\chi^2= 16.158$ (P = 0.001)
4	NMRSD	9	24	54	13	
5	MA	17	26	34	23	$\chi^2= 10.759$ (P = 0.013)
5	NMRSD	19	30	34	16	
6	MA	17	29	29	25	$\chi^2= 34.370$ (P = <0.001)
6	NMRSD	15	41	29	15	
7	MA	12	28	33	28	$\chi^2= 35.045$ (P = <0.001)
7	NMRSD	12	40	30	17	
8	MA	12	28	31	29	$\chi^2= 22.842$ (P = <0.001)
8	NMRSD	13	35	33	19	
10	MA	40	27	21	12	$\chi^2= 38.128$ (P = <0.001)
10	NMRSD	55	27	13	5	

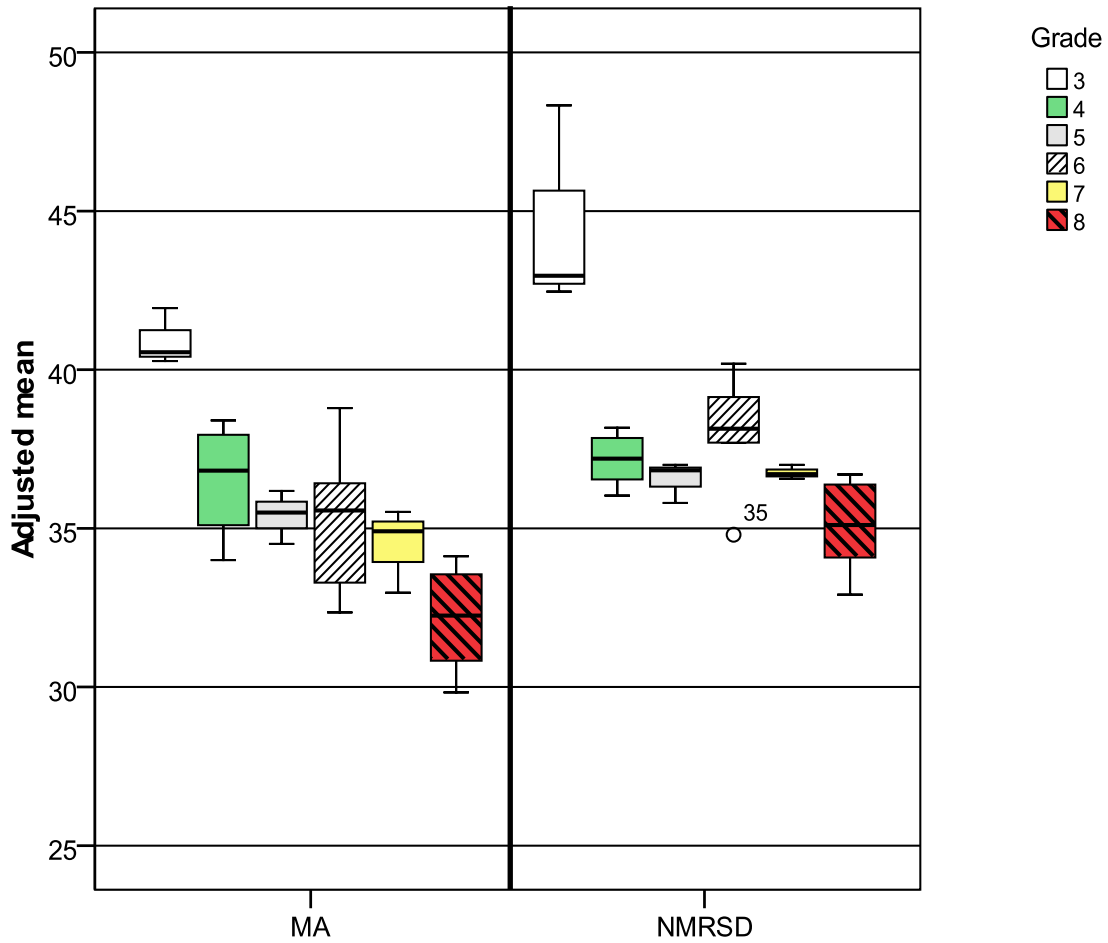


Figure 1. MA and NMRSD mean MCAS scores, by grade, averaged for years 2002-03 to 2007-08.

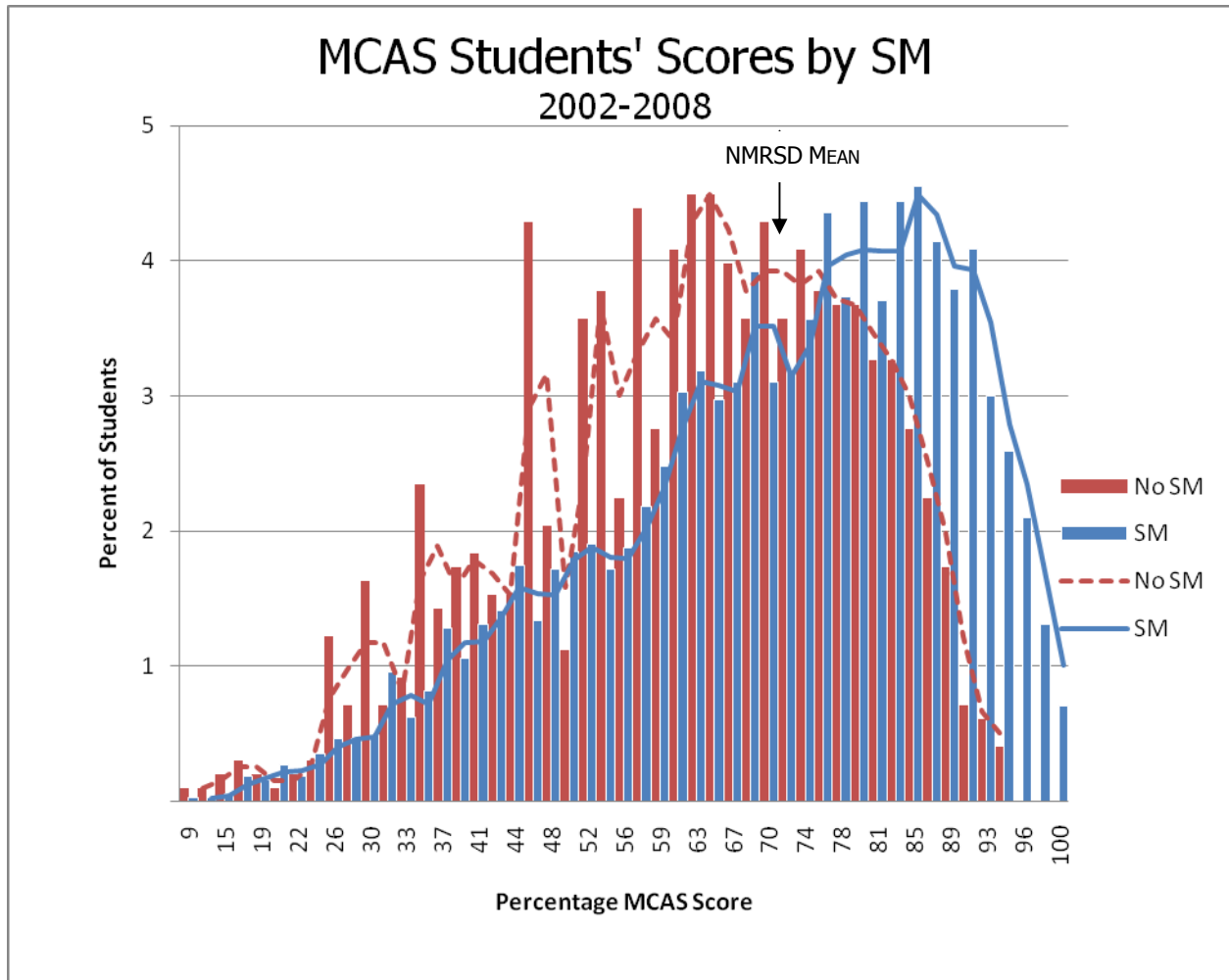


Figure 2. Percentage of Students with Coded MCAS Score, with and without SM (2002-08). Note the greater numbers of SM students with higher scores.

Table 3. Means by CSM History for V1						
Grade-Score	CSM 04-2002	CSM 06-2004	CSM 08-2006	Mean	Std. Dev.	N
4th Grade MCAS Math RAW	0	0	0	25.5	7.234	4
	0	0	2	34.4	8.484	142
	0	1	1	11.5	7.778	2
	0	1	3	37.1	8.949	54
	0	2	3	36.0	.	1
	0	2	4	35.7	8.351	78
	1	1	1	19.0	.	1
	1	1	3	30.3	17.898	3
	1	2	2	28.0	.	1
	1	2	4	32.1	12.335	7
6th Grade MCAS Math RAW	0	0	0	21.8	8.732	4
	0	0	2	39.0	8.173	142
	0	1	1	19.0	7.071	2
	0	1	3	40.5	9.691	54
	0	2	3	50.0	.	1
	0	2	4	41.0	8.062	78
	1	1	1	32.0	.	1
	1	1	3	39.3	8.737	3
	1	2	2	20.0	.	1
	1	2	4	39.1	9.839	7
8th Grade MCAS Math RAW	0	0	0	19.0	6.733	4
	0	0	2	37.0	9.886	142
	0	1	1	12.5	2.121	2
	0	1	3	39.9	10.924	54
	0	2	3	49.0	.	1
	0	2	4	38.2	9.778	78
	1	1	1	26.0	.	1
	1	1	3	40.0	10.817	3
	1	2	2	19.0	.	1
	1	2	4	35.1	10.511	7
1	3	5	44.8	8.684	14	

We see the value of SM in Table 3 above: for example, students with no SM had a mean score of 25.5 in 4th grade, 21.8 in 6th grade and 19 in 8th grade. Students with no SM in 2002, 1 year of SM in either 2003 or 2004 and SM in each of 2005 and 2006 (i.e., 0, 1, 3) scored 37.1 in 2002, 40.5 in 2004 and 39.9 in 2006.

Table 4. MCAS Mean Percent Correct by Category
8th grade, 2006

CSM, 8th grade 2006	MCAS Math Raw	MCAS Number Sense	Patterns, Relationships & Algebra	Geometry	Measurement	Data, Statistics & Probability
0	35.00	37.50	41.75	28.75	14.25	40.75
1	31.33	26.00	33.33	33.33	24.00	39.33
2	68.33	64.22	69.33	63.31	68.13	75.68
3	74.26	71.05	76.17	71.69	76.10	76.47
4	69.40	64.88	70.40	63.99	68.51	77.74
5	82.86	84.36	84.79	73.50	81.64	85.86

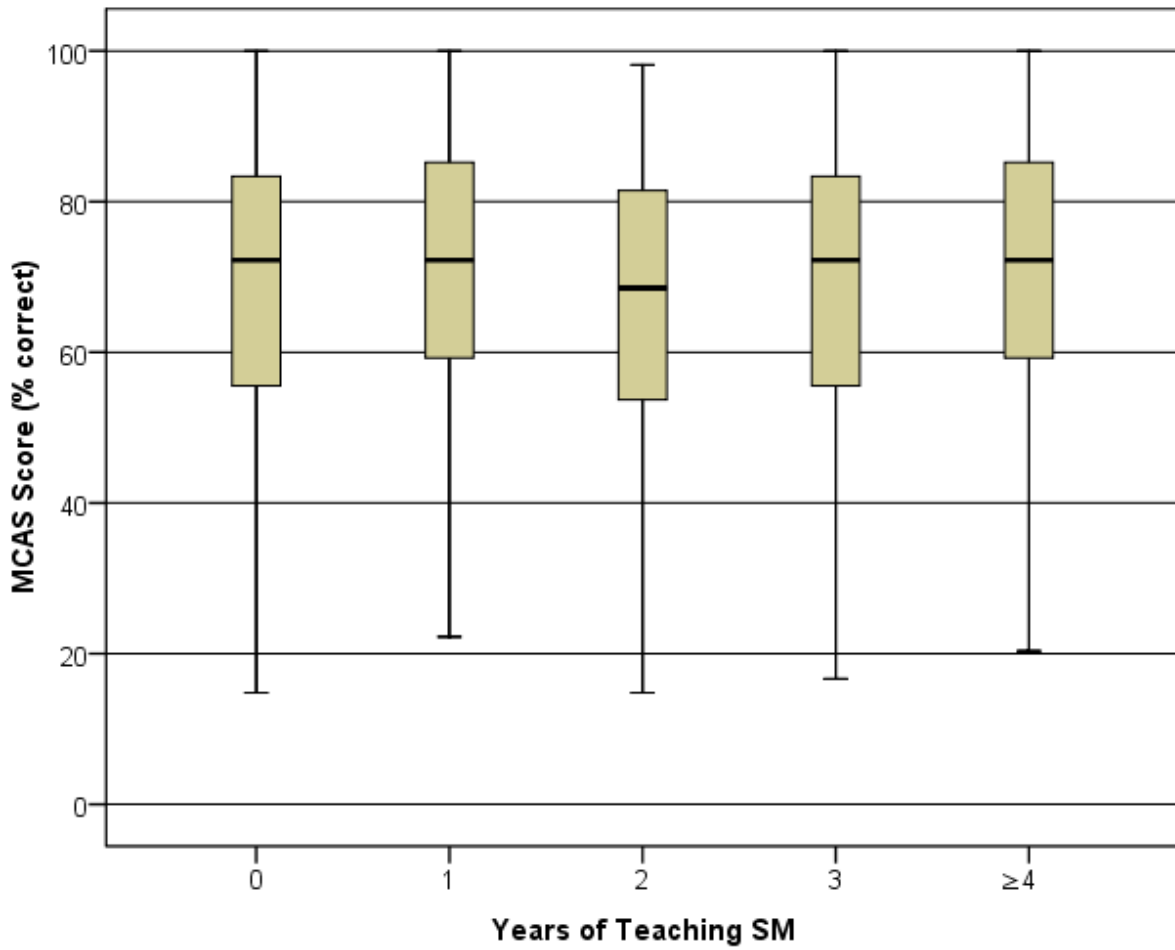


Figure 3. The mean percentage scores (2002-08) of all NMRSD students whose teachers' SM experience was 0 to 4 or more years) at the time.

SM=0		SM>0			Δ	
tchr	Mean	N	Mean	N	Total N	
18	43.94	17	38.59	216	233	-5.35
37	36.81	16	29.00	60	76	-7.81
38	34.30	20	35.26	82	102	0.96
48	34.78	37	36.84	283	320	2.06
56	37.75	16	38.14	65	81	0.39
59	37.83	18	39.03	62	80	1.20
69	34.00	21	40.14	56	77	6.14
72	40.05	19	37.47	60	79	-2.58
76	30.21	19	38.73	41	60	8.52
85	37.25	8	38.61	70	78	1.36
89	34.86	14	36.21	192	206	1.35
94	38.52	21	41.15	33	54	2.63
99	37.80	65	38.12	255	320	0.32
105	38.43	30	38.93	302	332	0.50
110	36.96	46	42.22	383	429	5.27